Symmetry and Relativity
An Introduction

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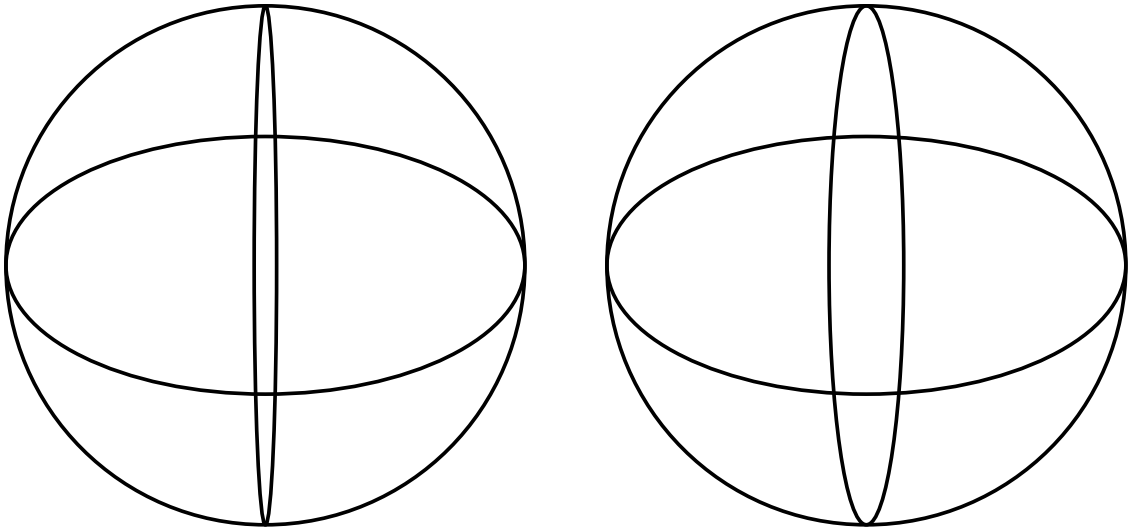
talk at NIH, June 03

Symmetry of objects

= invariance under transformations



Examples: empty space and time as "objects"

homogeneity of space

isotropy of space

homogeneity of time

Transformations of internal states

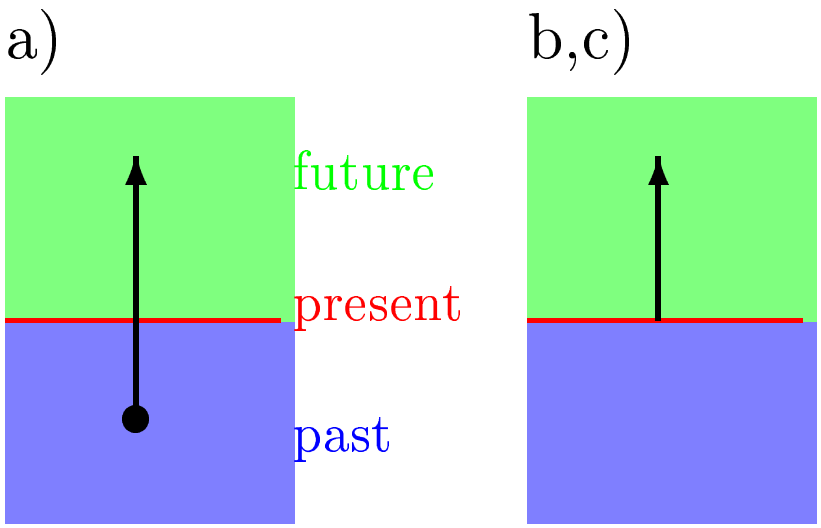
Rotate the elementary magnets (electron's spin) in a ferromagnet without rotating the whole magnet

Laws of physics

- equations of motion
  determine future from present (**Dynamics**).
  Example: Newton $\dot{p} = F$
- Constraints on state at one time
  Example: magnetic field is source free
- Conservation laws,
  follow from equations of motion
- Properties of large ensembles
  (Thermodynamics...)

Dynamics

The most important laws are equations of motion. They are determined by a Hamiltonian H.



a) **Dynamics general:** Past and present determine future

b) **Markovian Dynamics:** The present determines the future - what is relevant of the past is stored in a presently accessible memory

c) **Stochastic Markovian Dynamics:** randomness enters, predict probability of a certain future

solution of equations of motion $\mapsto$ history from initial state

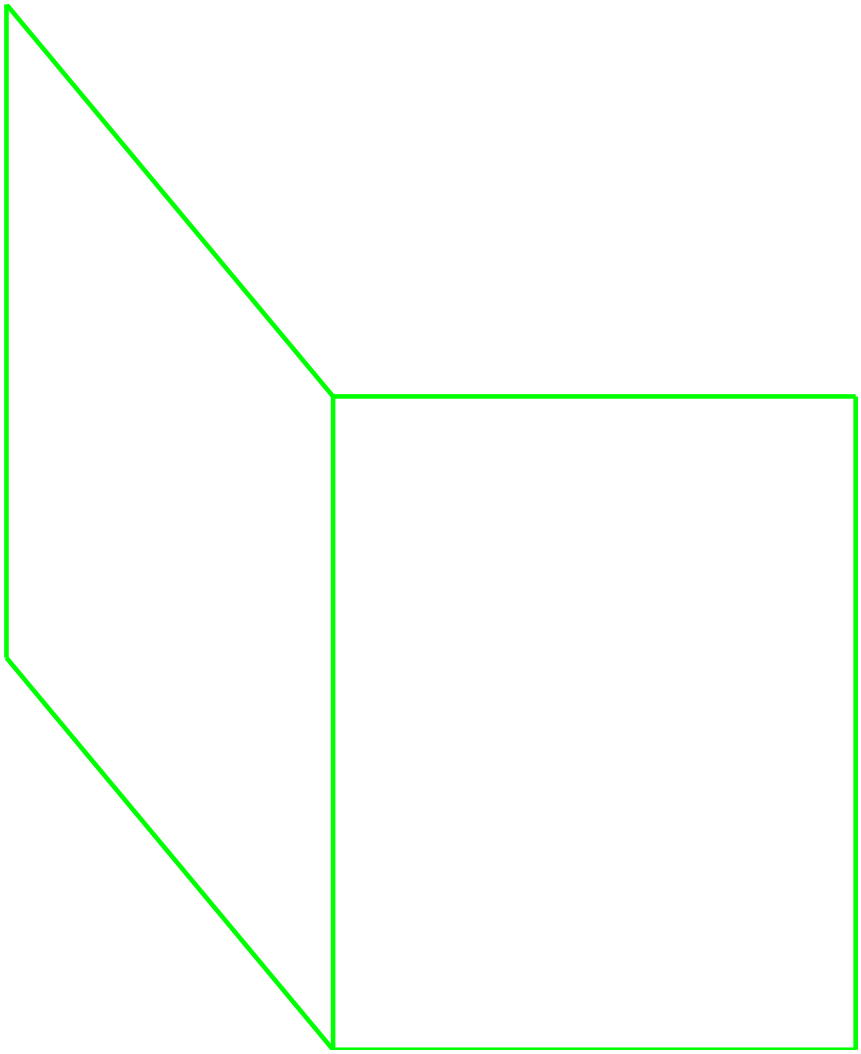
Symmetries of laws

Transform all objects (=the state) whether symmetric or not. Then afterwards the same laws are valid for them

Consequence: a possible history is transformed into another possible history

Many symmetries of laws are symmetries of space

Example: Throw a stone in different directions.



(draw parabolas by hand)

E. Noethers principle

Every continuous symmetry gives rise to a conservation law

Homogeneity of time $\mapsto$ conservation of energy

Homogeneity of space $\mapsto$ conservation of momentum

Isotropy of space $\mapsto$ conservation of angular momentum

Explain for chemists from basic Quantum Mechanics:

Hamiltonian H effects translation in time:

$$i\hbar\frac{d}{dt}L_z = L_zH - HL_z$$

z-component L_z of angular momentum effects rotation around z-axis. Therefore change of Hamiltonian under rotations is

$$HL_z - L_zH = 0$$

by isotropy of space

Symmetry of Ground states and its breaking

In quantum mechanics (at temperature 0) the ground states is the state of minimal energy

In statistical mechanics (at temperature T) the ground state is the state of minimal free energy

If the basic laws - e.g. the Hamiltonian - have a symmetry, it does not mean that ground states have the symmetry. IF NOT, this is called

spontaneous symmetry breaking

Illustrate this with mexican hats.

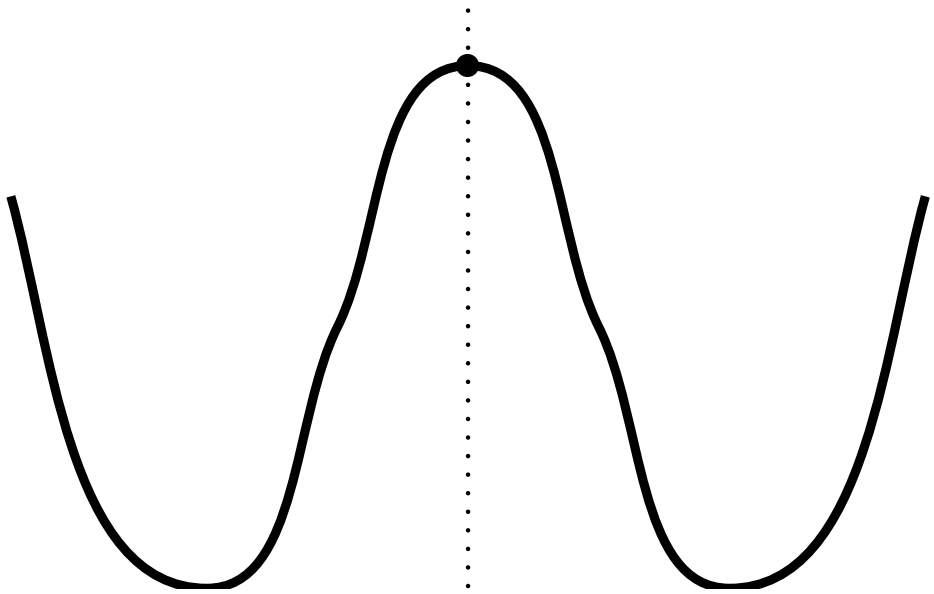


Figure 1: Mexican hat is symmetric under reflection

1. Marble rolling down from middle,
   ends up in one of the minima:
   asymmetrical situation

2. Consider Ferromagnet:
   Many elementary magnets $\uparrow$ or $\downarrow$
   spontaneous magnetization
   $M = N_{up} - N_{down} \neq 0$
   Free energy A depends on M given T
   Ground state $A(M) = min$
   If A is mexican hat: asymmetrical situation

Local transformations and local gauge invariance as symmetry of laws

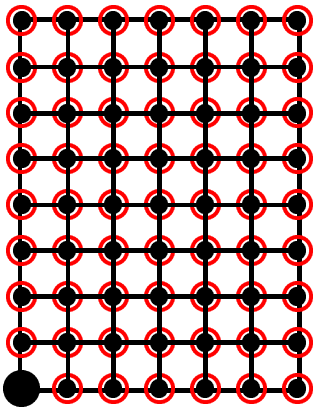


Figure 2: Agents at every node communicate through links which house interpreters

Local (gauge) transformations: Every agent may switch his language independently, and interpreters adjust to that.

Invariant content of the communication is "gauge invariant"

Laws of the trade must be gauge invariant.

Two more examples of invariants: *horse, brothers*

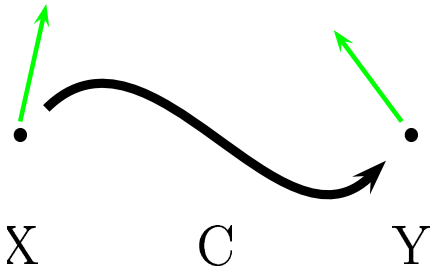
The physical principle behind local gauge invariance

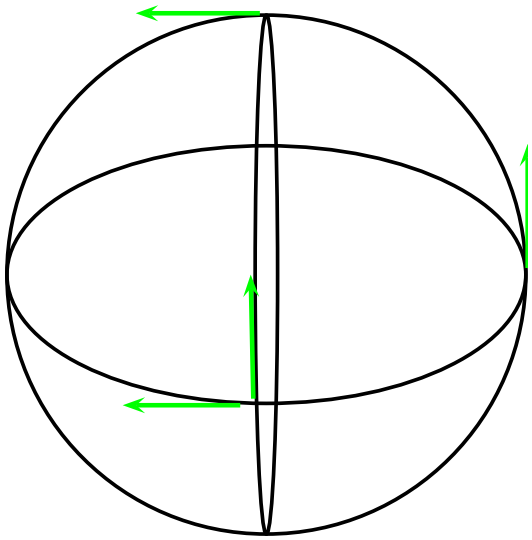
Fundamental physical theories are gauge theories, their laws are gauge invariant in the above sense.

19th century: No action at a distance (Faraday)

20th century: No information at a distance. Signals propagate along some channels, and are influenced by what they meet. Example:

The essence of general relativity: Parallel transport of vectors





Geometry

Points

Straight lines

Circle = points of equal distance from a reference point

General relativity is a geometric theory:

straight line: tangent vectors are parallel when transported along the line.

distance is defined

Sokrates and Einstein are relatives

Some ignorance is bliss: To know that you dont know and cannot know, because there exists no meaningful statement which would answer the question.

Einsteins equations are uniquely determined by the requirement that they should make sense

(be invariant under local transformations)

Examples from Elementary particle physics

Matter is made of basic constituents, quarks and leptons. There are six types of quarks: d, s, b and u, c, t.

d, s, b are alike except for vastly different masses. similarly for u, c, t

There are six types of leptons, e, μ, τ and neutrinos ν_e, ν_μ, ν_τ

Stable matter involves d and u-quarks in the protons and neutrons of atomic nuclei, and electrons e around them.

Theory: Gauge theory called *standard model*

Higgs mechanism

Spontaneous breaking of symmetries that come from local gauge invariance, with resulting spontaneous magnetization.

$$\text{Einstein:} \qquad m^2 = \frac{1}{c^4}\left[E^2 - p^2c^2\right]$$

If the energy E receives contributions from spontaneous magnetization (not just kinetic energy from momentum p) then mass $m \neq 0$.

The masses of all (?) elementary particles originates from spontaneous breaking of the basic gauge invariance of the standard model.

(? including right handed neutrinos ?)

LMP-theory: We now understand that the vastly different masses of quarks comes from spontaneous breaking of another gauge symmetry which interchanges d, s, b and u, c, t.

Functionality as Structure

Put yourself in the position of a supreme being who sees the world at all times at once as a big 4-dimensional system called a drama.

K. Vonnegut: *Men in 4 dimensions are a kind of luminous spaghetti, with a baby shape at one end, and elderly at the other*

Given the initial state at an initial time, specific dramas take place with a certain probability, if there is randomness (stochastic dynamics)

It is of the type of an equilibrium statistical mechanics in 4 dimensions.

Functionality = structure of the drama

Concepts of equilibrium stat. mech. like spontaneous symmetry breaking become applicable to study of functionality.

Study life as a problem in equilibrium statistical mechanics in 4 dimensions:

Remember Kurt Vonnegut's spaghetti ...

Emergent behavior is an aspect of self organization, where composite objects form whose constituents behave in a coherent fashion. Flocks, herds and schools have been named among the prototypical examples

> *Schools of fish swimming in coherent array abruptly turn together with no leader guiding the group.*

This can be explained as restoration of spontaneously broken symmetry (under rotations) by propagating shock wave