From a general Theory of Complex Systems to Elementary Particles

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The universal structure of human thinking

Two different problems

1. What characterized a truly fundamental physical theory?
2. On what to build a general theory of all kinds of complex systems in physics, chemistry, sociology, linguistics, cognition,...

leads to the same

Question: What is the minimal *a priori* structure needed to build on it a theory of the world?

Answer: the universal structure of human thinking.

- A physical theory will be the more fudamental, the less structure *a priori* is assumed
- A theory of complex systems is the more general, the less structure

AXIOM: *Der human mind models the world as a network of relations between things or agents*

These models are called Systems.

Kinds of symbolic representations

verbal : humanities

quantitative : physics, natural sciences

structural : chemistry, linguistics, systems theory

Einstein's Principles

ar pushed to the extreme: No numerical description to begin with.

1. Principle of relativity or general covariace
2. Principle of equivalence

Einsteins relativity principle is a principle of absence of a priori *structure!*

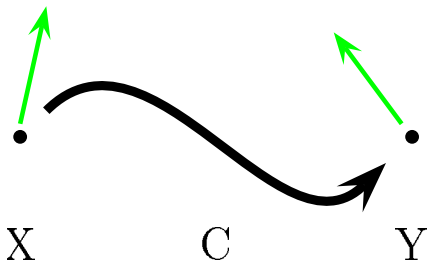
Before: Space had *a priori* structure, which defines the notion of a straight line.

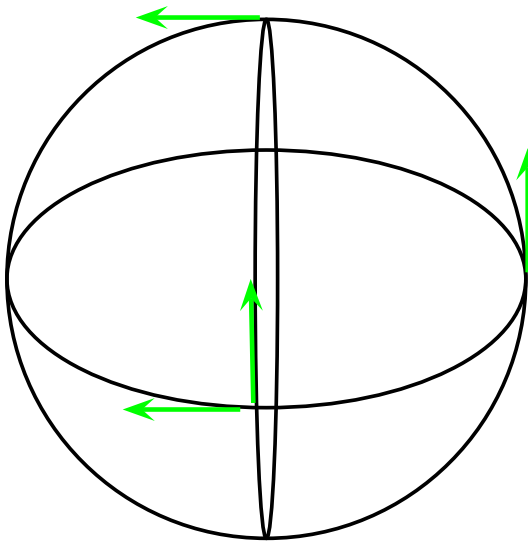
After: Parallel transport dynamically determined

But: Straight lines in the infinitesimally small are defined *a priori* - the principles are not pushed to their logical conclusion.

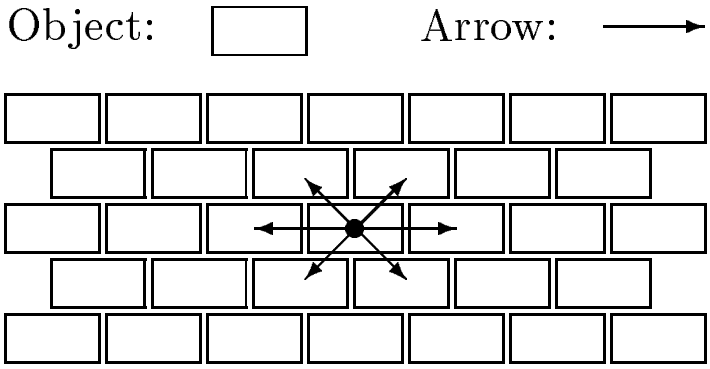
Hope: A sufficiently strong principle of general covariance should suffice to single out the truely fundamental physical theories

Paralleltransport of vectors in general relativity





How to describe Structure?



1. Structure is described as a network of relations f between objects X. The relations are represented as arrows, $f : X \mapsto Y,\ g : Y \mapsto Z$....

2. Each arrow f determines an arrow in the opposite direction, $f^* : Y \mapsto X$.
   husband$\leftrightarrow$ wife, brother$\leftrightarrow$ brother/ sister

3. Among the relations (arrows) is the identity ι_X of each object X with itself.

4. Arrows can can be composed, write $g \circ f : X \mapsto Z$.
   Some arrows are distinguished as fundamental(links), all others are composed of fundamental arrows.
   friend of a friend, father in law, nephew,...

Def: Frustration is present when several arrows exist between 2 given objects (*"path dependence"*).

Electrodynamics Gauge theories of elementary particles	field strength, e.g. electric , magnetic
Surfaces in space General relativity	curvature
Financial markets	arbitrage
Systems theory Spin glasses	frustration

Table 1: Frustration under different names

Locality:

Objects, which can be reached from X throuh=gh one link form a *neighborhood* of X, etc.

Links symbolize local interactions. The links pointing to an object are called its *valences*

How to build a theory of the world on so little *a priori* structure?

A: In two steps

1. Name things
2. Make assertions about named things

In fundamental physics the following things are named and investigated

- Space (im the sense of spacelike hypersurfaces in space-time) ($\Longrightarrow$ constraints of general relativity)
- Elektromagnetic fields and Yang-Mills-Fields
- Matter (Dirac Fields).
- In addition: What is a thing, what is life?

Explanation:

Distinguish two kinds of physical laws

1. constraints:$\Rightarrow$ Names
   constrain the state at one (and each) time
2. Local laws of motion$\Rightarrow$ predictions
   They should be meaningful for arbitrary systems:
   (*Universal Dynamics*.)

All fundamental physical laws obey nontrivial constraints.
Give names to things which obey certain constraints

$\Rightarrow$ Coordinate independence to the extreme (cp. Einstein)

$\Rightarrow$ no numerical description to begin with

$\Rightarrow$ Dynamics made of structural transformations

Gauge theories

Special case of the basic idea: Characterize systems of different kinds by properties of the relations (channels of communication), i.e. by constraints involving them.

Main Theorem: (From general Complex Systems to Gauge Theories) *Suppose all links are unitary*

$$forth \circ back = identity.$$

Interpret X as sites in space. Then we get all the structure of a lattice gauge theoy (on a possibly irregular lattice)

1. group G of local gauge transformatoions
   same for all X, independent of time under any (universal) dynamics
2. local color spaces Linear space Ω_X at each X
3. parallel transporters are linear maps $\Omega_X \mapsto \Omega_Y$
   $\circ$ is composition of maps

Local gauge group G_X consists of all arrows $g : X \mapsto X$. G_X is isomorphic for different X; $g^{-1} = g^*$. The gauge group is a constraint which characterizes a system.

Gauge transformations:

$$f : X \mapsto Y \quad \Rightarrow g_Y \circ f \circ g_X^{-1}$$

cp. Electrodynamics: $A_i(\mathbf{r}) \mapsto A_i(\mathbf{r}) + \nabla_i \Lambda(\mathbf{r})$,

Path $C : X \mapsto Y$ defines

$$A(C) = \int_C \mathbf{A}(\mathbf{r}) d\mathbf{r} \equiv f$$

Composition $\circ$ is $+$, $A(C) \mapsto A(C) - \Lambda(X) + \Lambda(Y)$

Corollary: Financial markets without fees are gauge theories because

$$\text{sale} \circ \text{purchase} = \text{identity}$$

Financial markets *sub specie aeternitatis* are nearly unfrustrated $\Longrightarrow$ Black Scholes formula for the price of options.

Communication: Exchange or sale/purchase, arrows = exchange rates.
Fees $\Longrightarrow$ Dissipation!

challenge to physicists: Invent gauge theories with dissipation.

Local dynamic made of structural transformations

Dynamics determines a time evolution $t \mapsto S_t$ of a system, given a system as initial state S_{t_0}. We consider discrete time steps τ.

The dynamics is local if each object X' of the system is descendent of (at least) one object X of S_t, and the links with target X' are made of links in a neighborhood.

"Drama" = System *sub specie aeternitatis* $S \supset S_t \ (\forall t)$

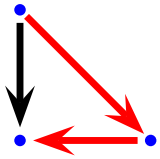
The possible local structural transformations are called *Enzymes*. Since they act somewhere, they are considered as objects. cp. biology.

Universal dynamics: defined for every system.

Only three types of local transformations (& new links)

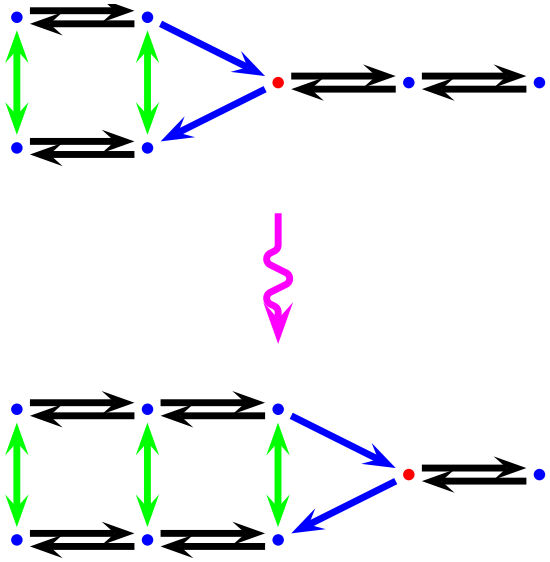
Motion :

Composite arrows become fundamental



Growth :

Copy objects, add adjoint links



Cognition :

Generate links between composite objects (systems) with matching internal structure.

How does one make acquaintances? Similarly in 3 ways.

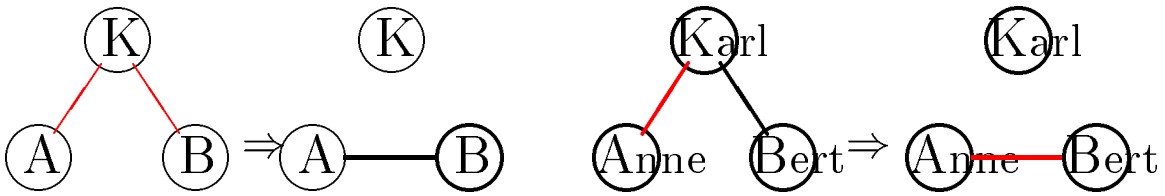


Figure 1: Catalysis in der chemistry (and elsewhere). Catalyst K binds molecules A und B. First a substrate-enzyme-complex is formed, where A and B are bound to K. Then the composite arrow between A and B is transformed into a fundamental arrow.

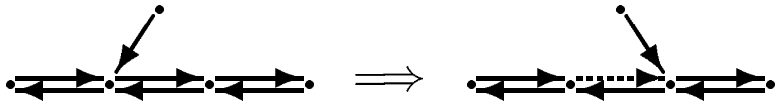


Figure 2: Interpretation of motion as a transformation of a composite arrow into a fundamental one. The upper point represents a particle, the others space points. The link from particle to space point x represents the relation "*at*". Motion occurs when the composite arrow made from relation b of the particle to its former position and the relation of this space point to its neighbor becomes fundamental.

Maxwell, Yang-Mills, Dirac Equation

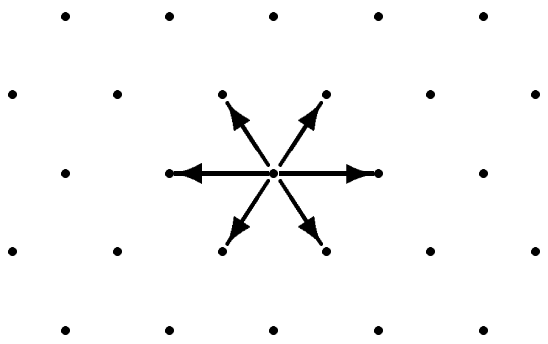


Figure 3: triangular lattice

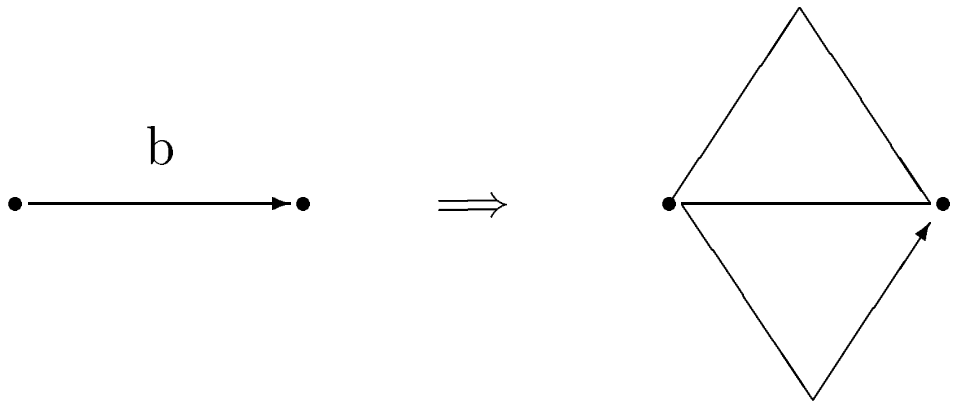


Figure 4: The universal equation of motion of fundamental physics. A composite arrow becomes fundamental. The symbol $\Longrightarrow$ symbolizes the action of one time step. The Dirac equation is a special case (one corner $\bullet$ at ∞)

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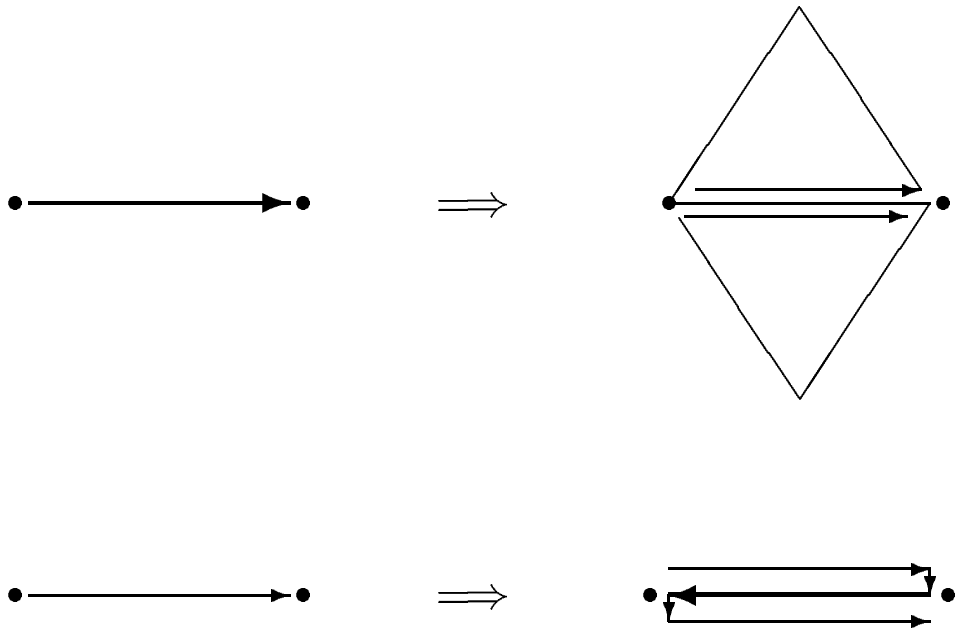

Figure 5: Maxwell equation of electrodynamics, Yang Mills equations of elementary particle physics. In the presence of Dirac matter, the triangles can have a tip at ∞. 2 kinds of links from $\mathbf{A}$, $\mathbf{E}$.

Explanation: Links composition $\circ$ is $+$.

$$\bullet \to \bullet \quad = \quad \int \mathbf{A} d\mathbf{x} \approx \mathbf{A}(x) d\mathbf{x} \qquad \text{hence} \qquad (1)$$

$$\Delta \quad = \quad \oint_{\Delta} \mathbf{A} d\mathbf{x} = \int_{F:\partial F = \Delta} \mathbf{B} d\mathbf{f} \qquad (2)$$

Summation over triangles touching $\to$ gives $\nabla \times \mathbf{B} \cdot$ area

Consistency of this equation implies conservation of electric charge!

The indestructibility of matter results from the structural description - no separate postulate

Constraints: Gauss' law, gauge group $U(1)$.

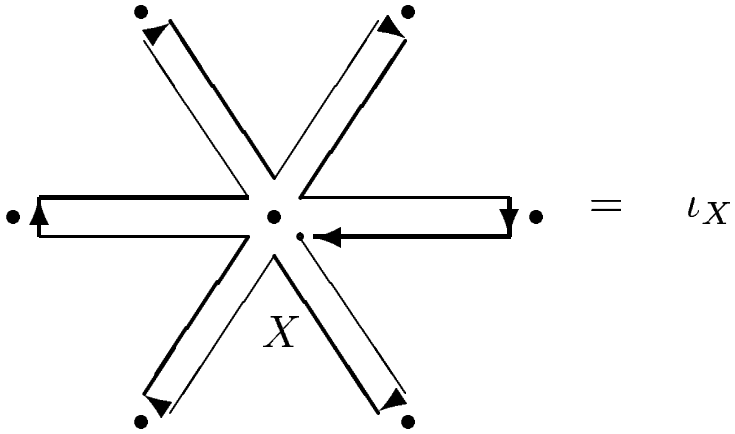

Figure 6: Gauss' law for electrodynamics, Yang Mills theory and general relativity. With Dirac matter, one of the tips is at ∞.

Geometrical interpretation of the Higgs

The double sheeted world

One sheet carries lefthanded matter, the other right handed matter (Weyl Spinors).
Higgs fields are the parallel transporters in between

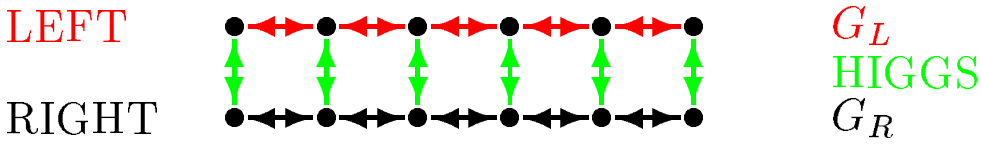

Figure 7: Systemtheoretic Higgs-world: Gauge group $[SU(3) \times SU(2) \times U(1)]_L \times [SU(3) \times SU(2) \times U(1)]_R$ is broken to $SU(3)_{diag} \times U(1)$ by Higgs. No anomaly. Expect strong intermediate vector boson (color octet)

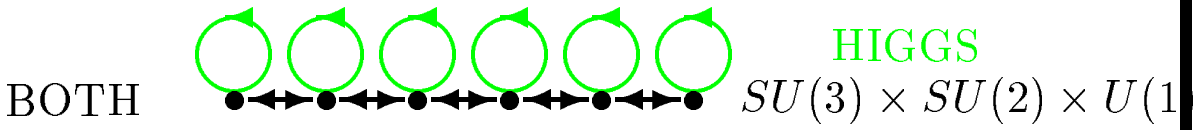

Figure 8: The points could be projections of a 5-th dimension, with domain wall fermions on its 4-dim. boundaries.

For contrast:Structural description of Logic

A logical system has very few unitary relations. They are interpreted as negation,

$$negation \circ negation = identity$$

Definition: A logical system has
objects interpreted as propositions
links $\longrightarrow$ interpreted as *excludes*, as *negation* if unitary
composite objects $A|B$, interpreted as *neither* A *nor* B,
with links

$$A \longleftrightarrow A|B \longleftrightarrow B \ .$$

The arrow $A \longleftrightarrow A|A$ is unitary.

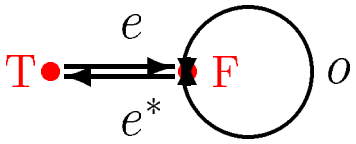

Figure 9: Logical archetype: e unitary, $o \circ o = o = o^*$ etc.

Note on adjoint:

If A *excludes* B, then B *excludes* A.

Logical Theorem *Every structure preserving map of a logical system into the logical archetype assigns truth values in accord with the rules of logic.*