How similar are general Complex Systems?

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Es ist überhaupt nichts so wichtig im Leben, als genau den Standpunkt auszumitteln, aus welchem die Dinge aufgefaßt und beurteilt werden müssen , und an diesem festzuhalten, denn nur von *einem* Standpunkt aus können wir die Masse der Erscheinungen mit Einheit auffassen, und nur die Einheit des Standpunkts kann uns vor Widersprüchen schützen.

Claus von Clausewitz

Complex systems are all similar in the way we think about them:

The human mind thinks about relations between things or agents

[*Das Mathematische:*] Damit wird im Grundriß vorgezeichnet, wie jedes Ding und jede Beziehung jedes Dings zu jedem Ding gebaut ist. *Martin Heidegger*

One language for all sciences

Complex systems are of interest in many sciences

Natural sciences: Physics, Chemistry, Biology, Biochemistry, Informatics, experimental Psychology,...

Social sciences: Sociology, political science, research on conflicts, Economy, Finance,...

Humanities: Linguistics, Philosophy (epistemology, ontology), Theology ...

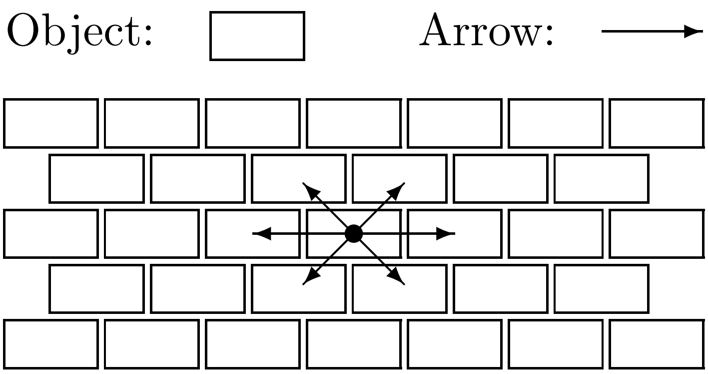
To see the forest and not only the trees we need a universal language (symbolic representation). Until now

verbal : Humanities and Social Sciences

quantitative : Exact Sciences, esp. Physics

structural: Chemistry, Linguistics, Systems theory

How to describe Structure?



1. Structure is described as a network of relations f between objects X. The relations are represented as arrows, $f : X \mapsto Y$.

2. Each arrow f determines an arrow in the opposite direction, f^*.
   husband↔ wife, brother↔ brother/ sister

3. Among the relations (arrows) is the identity of each object with itself.

4. Arrows can can be composed, write $g \circ f$. Some arrows are distinguished as fundamental(links), all others are composed of fundamental arrows.
   friend of a friend, father in law, nephew,...

Def: Frustration is present when several arrows exist between 2 given objects (*"path dependence"*).

Electrodynamics Gauge theories of elementary particles	field strength, e.g. electric , magnetic
Surfaces in space General relativity	curvature
Financial markets	arbitrage
Systems theory Spin glasses	frustration

Table 1: Frustration under different names

Locality:

Objects, which can be reached from X throuh=gh one link form a *neighborhood* of X, etc.

Links symbolize local interactions. The links pointing to an object are called its *valences*

How to build a theory of the world on so little *a priori* structure?

A: In two steps

1. Name Dinge
2. Make assertions about named things

In fundamental physics the following things are named and investigated

- Space (im the sense of spacelike hypersurfaces in space-time) ($\Longrightarrow$ constraints of general relativity)
- Elektromagnetic fields and Yang-Mills-Fields
- Matter (Dirac Fields).
- In addition: What is a thing, what is life?

Explanation:

Distinguish two kinds of physical laws

1. constraints:$\Rightarrow$ Names
   constrain the state at one (and each) time
2. Local laws of motion$\Rightarrow$ predictions
   They should be meaningful for arbitrary systems:
   (*Universal Dynamics*.)

Representation theorem 1: Every system can be represented as a communication network as follows:

Objects X : associate (input) spaces Ω_X

Arrows f: $X \mapsto Y$ become maps $f : \Omega_X \mapsto \Omega_Y$.

Channels of communication

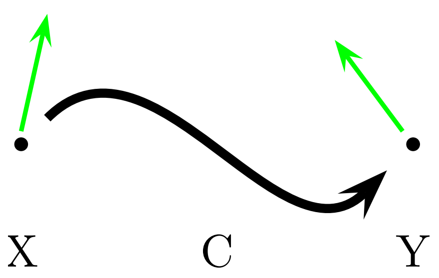
All fundamental physical laws obey nontrivial constraints.
Give names to things which obey certain constraints

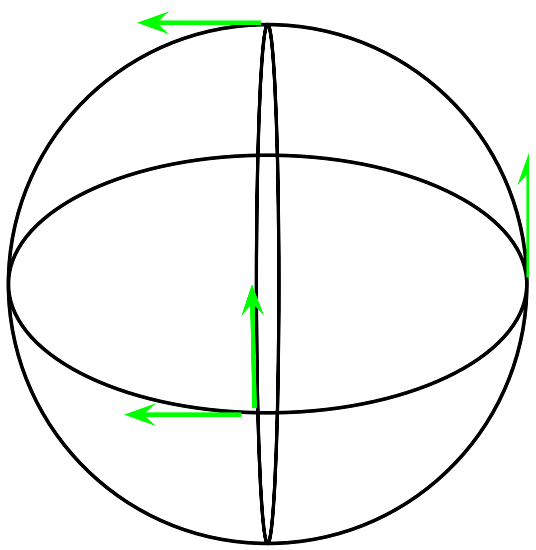
$\Rightarrow$ Coordinate independence to the extreme (cp. Einstein)

$\Rightarrow$ no numerical description to begin with

$\Rightarrow$ Dynamics made of structural transformations

Parallel transport von Vectors in general relativity





Gauge theories

Special case of the basic idea: Characterize systems of different kinds by properties of the relations (channels of communication), i.e. by constraints involving them.

All fundamental physical theories are gauge theories.

Gauge theories are characterized by *unitary relations*

$$\text{forth} \circ \text{back} = \text{identity}.$$

Gauge group G_X consists of all arrows $g : X \mapsto X$. G_X is independent of X; $g^{-1} = g^*$. The gauge group is a constraint which characterizes a system.

Gauge transformationen:

$$f : X \mapsto Y \quad \Rightarrow g_Y \circ f \circ g_X^{-1}$$

cp. Electrodynamics: $A_i(\mathbf{r}) \mapsto A_i(\mathbf{r}) + \nabla_i \Lambda(\mathbf{r})$,

Path $C : X \mapsto Y$ defines

$$A(C) = \int_C \mathbf{A}(\mathbf{r}) d\mathbf{r} \equiv f$$

Composition $\circ$ is $+$, $A(C) \mapsto A(C) - \Lambda(X) + \Lambda(Y)$

Representation theorem 2: In Gauge theories Ω_X can be taken as a representation space for the gauge group.

X space points $\longrightarrow$ lattice gauge theory.

Additional example: theory of financial markets without fees

Communication: Exchange or sale/purchase, arrows = exchange rates.

Fees:

$$\text{sale} \circ \text{purchase} \neq \text{identity}$$

Dissipation!

Local dynamic made of structural transformations

Dynamics determines a time evolution $t \mapsto S_t$ of a system, given a system as initial state S_{t_0}. We consider discrete time steps τ.

The dynamics is local if each object X' of the system is descendent of (at least) one object X of S_t, and the valences of X' are made of links in a neighborhood.

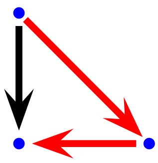
The possible local structural transformations are called *Enzymes*. Since they act somewhere, they are considered as objects. cp. biology.

Universal dynamics: defined for every system.

Only three types of local transformations (& new links)

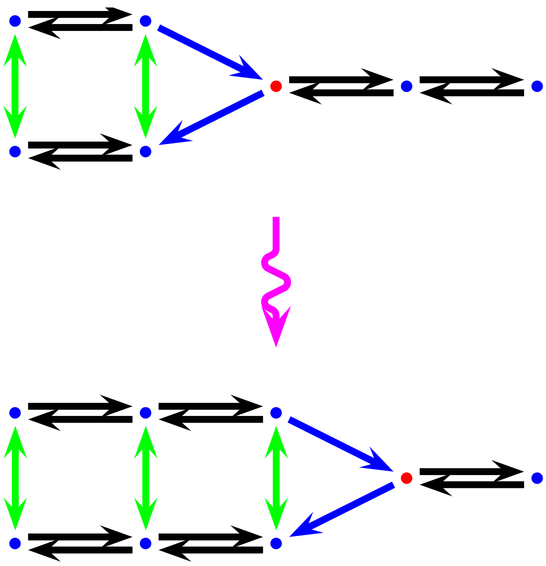
Motion :

Composite arrows become fundamental



Growth :

Copy objects, add adjoint links



Cognition :

Generate links between composite objects (systems) with matching internal structure.

How does one make acquaintances? Similarly in 3 ways.

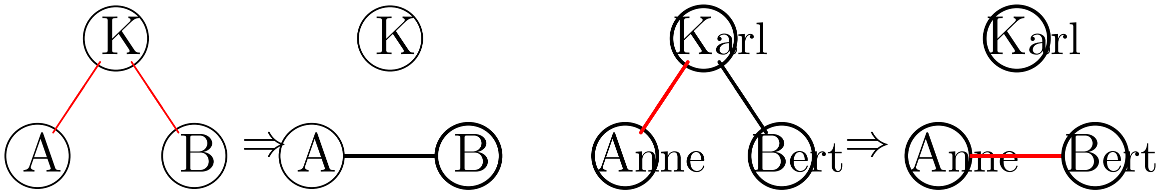


Figure 1: Catalysis in der chemistry (and elsewhere). Catalyst K binds molecules A und B. First a substrate-enzyme-complex is formed, where A and B are bound to K. Then the composite arrow between A and B is transformed into a fundamental arrow.

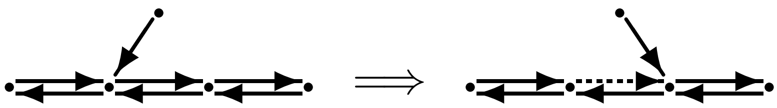


Figure 2: Interpretation of motion as a transformation of a composite arrow into a fundamental one. The upper point represents a particle, the others space points. The link from particle to space point x represents the relation "*at*". Motion occurs when the composite arrow made from relation b of the particle to its former position and the relation of this space point to its neighbor becomes fundamental.

What is complexity?

The behavior of a complex system cannot be understood by examination of subsystems which contain few of its constituents.

Counter example: dilute gas (example: critical point)

arbitrarily accurate free energy from consideration of 1,2,3... particles

How to understand complex systems?

By multiscale analysis. cp. Hakens "*slave principle*"

In each step certain subsystems are selected and declared composite objects. Fundamental arrows between them are constructed, and a dynamics of the composite objects is deduced (by a process of complexity reduction) from that of its constituents. (Example: biol. cells interact with environment only through membrane proteins)

If the internal structure is ignored, one obtains an effective theory NB: One desires a systematic universal procedure to identify composite objects - e.g. evolution by natural selection.

This involves a structure-detection problem (cognition)

Two fundamental notions:

Emergence: Nonlocal phenomena spring from local causes. **Examples:**

Elektromagnetic waves , radiated from antenna.

forest fire , from cigarette butt.

global world changes , from local causes

- Assassination in Sarajevo 1914
- In the year 1280 the Jewish scholar Abraham Abulafia travelled to Rome to convert the pope. Suppose it had worked...

Replication fork (s.above) induces global copy process

Self organization:

Emergence of composite objects (subsystems) whose constituents behave in a coherent fashion (e.g. move together, collect taxes,...) **Examples:**

Institutions of human society: They can form spontaneously (example: Clubs with very selective membership as formed in the recent self organization of international financial markets)

Fibres of a cell's sceleton aggregate spontaneously (Actin)

Maxwell, Yang-Mills, Dirac Equation

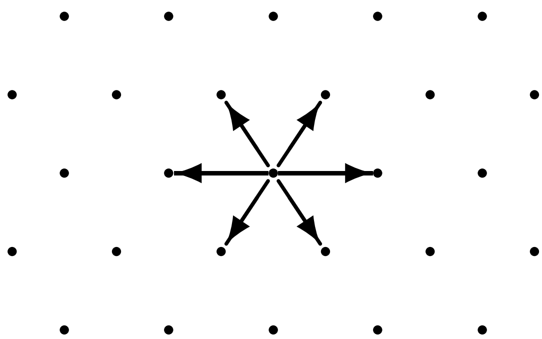


Figure 3: triangular lattice

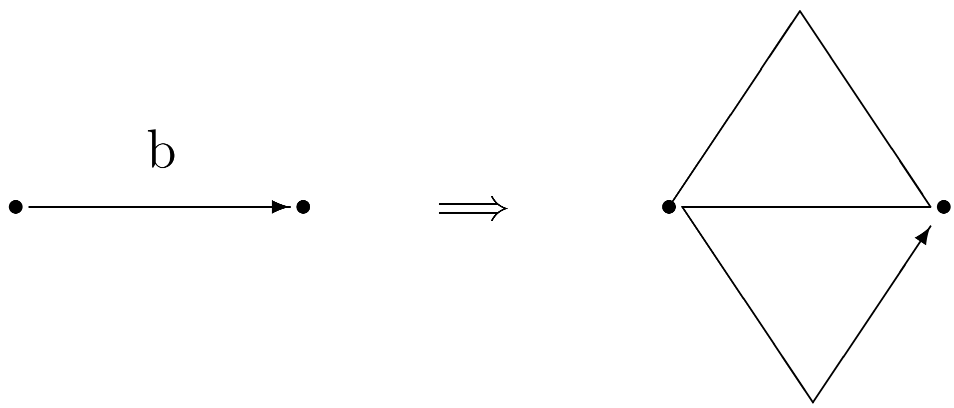


Figure 4: The universal equation of motion of fundamental physics. A composite arrow becomes fundamental. The symbol $\Longrightarrow$ symbolizes the action of one time step. The Dirac equation is a special case (one corner $\bullet$ at ∞)

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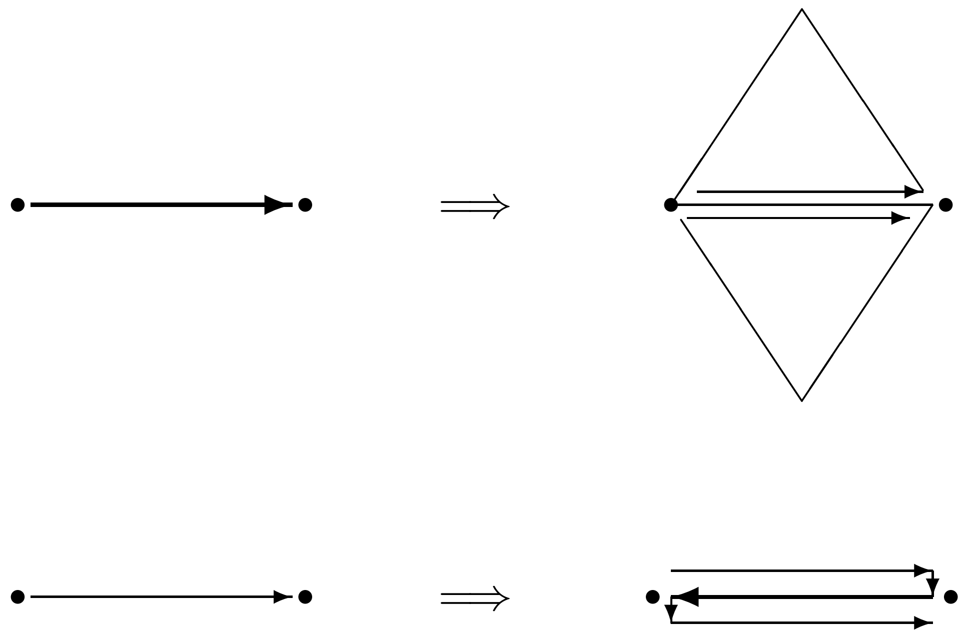


Figure 5: Maxwell equation of electrodynamics Yang Mills equations of elementary particle physics. In the presence of Dirac matter, the triangles can have a tip at ∞. 2 kinds of links from $\mathbf{A}$, $\mathbf{E}$.

Explanation: Links composition $\circ$ is $+$.

$$\bullet \rightarrow \bullet \quad = \quad \int \mathbf{A} d\mathbf{x} \approx \mathbf{A}(x) d\mathbf{x} \qquad \text{hence} \tag{1}$$

$$\Delta \quad = \quad \oint_{\Delta} \mathbf{A} d\mathbf{x} = \int_{F:\partial F = \Delta} \mathbf{B} d\mathbf{f} \tag{2}$$

Summation over triangles touching $\rightarrow$ gives $\nabla \times \mathbf{B} \cdot$ area

Consistency of this equation implies conservation of electric charge!

The indestructibility of matter results from the structural description - no separate postulate

Constraints: Gauss' law, gauge group $U(1)$.

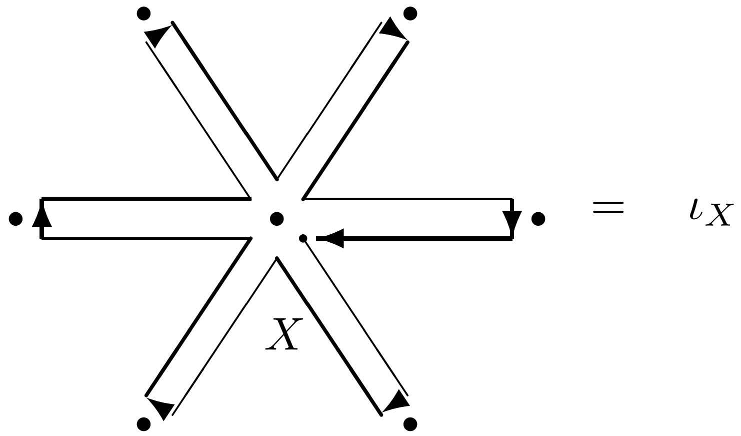


Figure 6: Gauss' law for electrodynamics, Yang Mills theory and general relativity. With Dirac matter, one of the tips is at ∞.

Geometrical interpretation of the Higgs

The double sheeted world

One sheet carries lefthanded matter, the other right handed matter (Weyl Spinors).
Higgs fields are the parallel transporters in between

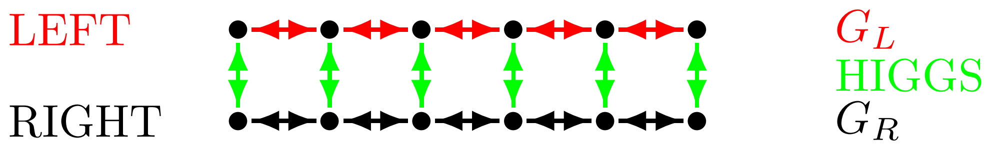


Figure 7: Systemtheoretic Higgs-world: Gauge group $[SU(3) \times SU(2) \times U(1)]_L \times [SU(3) \times SU(2) \times U(1)]_R$ is broken to $SU(3)_{diag} \times U(1)$ by Higgs. No anomaly. View with decreased resolution:

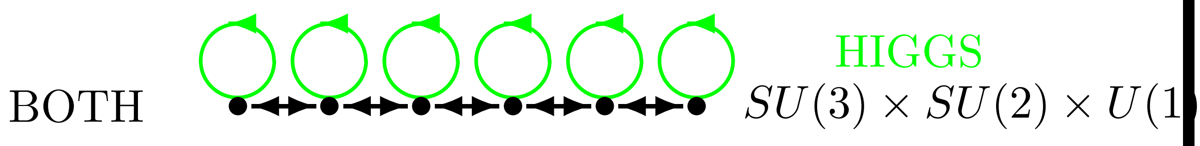


Figure 8: The points could be projections of a 5-th dimension, with domain wall fermions on its 4-dim. boundaries.

Enzymatic computation. Growth and evolution

basic idea: All kinds of computation are compose of local structural transformations - i.e. from the action of enzymes. (No synchronization $\mapsto$ no Turing machine.) cp. biology!

Example:

Arithmetics: + operation

$$\begin{array}{r} 4539 \\ +327 \\ \hline 1 \\ \hline ...6 \end{array}$$

This is fusion of two systems by a zipper like *local* operation.

$$\begin{array}{rcl} 4 \to 5 \to 3 \to 9 & \to & \oplus = \\ 3 \to 2 \to 7 & \nearrow & \\ 4 \to 5 \to 3 \to \oplus & \to & 6 \\ 3 \to 2 \to \overset{\uparrow}{\oplus} & \leftarrow & 1 \end{array}$$

Could be coded in DNA: e.g. $0 = AAT$, $1 = ATA$, addition-enzyme $\oplus$

$ATA \oplus AAT = AAT \rightarrow ATA$ etc.

NB: 0,1 are systems with different internal structure,

Algebraic manipulation programs e.g. MATHEMATICA use local structural transformations on trees extensively.

Iterative optimization , e.g. Solution of systems of equations.

Monte Carlo sweeps e.g. in lattice gauge theory

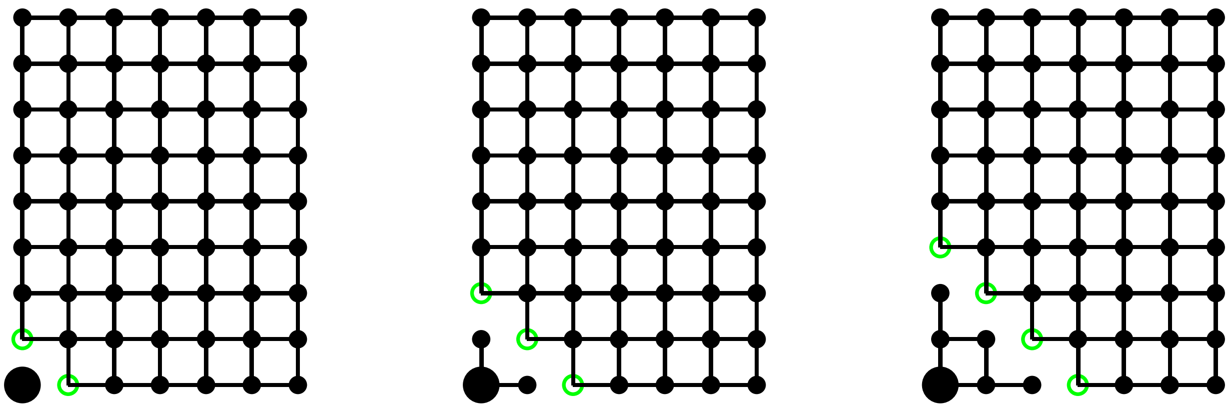
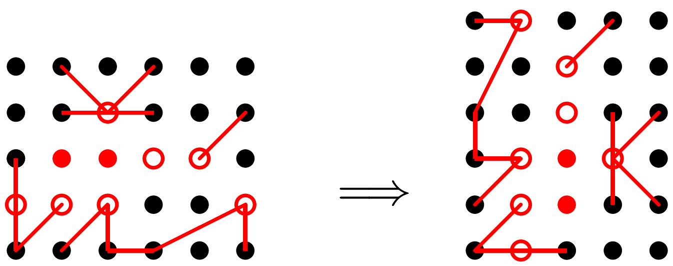


Figure 9: Shock fronts traveling through a crystal

Matching of structure :

shock-front-matching



Growth and evolution processes

Mutation=modification of constituents,

Cross over: $ACB + A'CB' \Rightarrow ACB' + A'CB$.

Natural selection: requires a cost functional

Examples:

1. Intercalary regeneration of portions of cockroach legs (tibia), depends on morphogen concentration
2. Mikroscopic simulations of the immune system (M. Meier-Schellersheim): e.g.
   - 8 cell Types, je 6....8 mechanisms , with 1 000 cells of each (e.g. killer T-cells)
   - 10 types of molecules, 5 000 molecules of each (e.g. cytokines)
   - 5 000 sweeps, 1 mechanism operates per cell and sweep

   mechanism = conditional action of an enzyme

Critical slowing down in optimization problems, Multiscale analysis

Standard problem: Minimization of a cost functional.

Examples:

Solution of equations

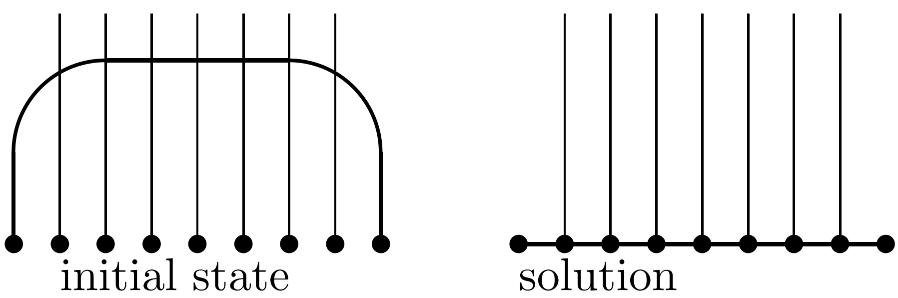
biology: Number of sexually mature offspring = max.

Extremal priciples in physics

Solution by iteration, see above

Problem: critical slowing down

Example: Solution of discretized linear partial differential equations, for instance 1-d Laplace eq. with 0 Dirichlet boundary conditions $\sum_i |y_i - y_{i+1}|^2 = min, \; y_0 = y_N = 0$



Remedy: Multiscale analysis: Adaptation on extended domains, e.g. by addition of suitable roof- functions $\frown$.

Special features of soial dynamical systems

financial markets, economy, political systems

cost functional=????

Idea: Degree of fulfillment of human needs.

Problem: Seems hopeless, because valuation is involved, and balance between incommensurable needs.

Remedy: Operative valuations are part of the description of a system at one time. They change dynamically. Self interaction, because adherence to values serves to satisfy a human need itself (to live conscious of ones worth)

Two kinds of variables:

Values (prices, exchange rates) gauge fields

Goods (facts of life) matter fields

In financial markets prices (exchange rates) are valuations, flow of assets are matter currents.

⇒ Gauge theory of financial markets without fees [Ilinsky]

Fees $\Longrightarrow$ nonunitary relations because

$$sale \circ purchase \neq identity.$$

Dissipation!